

THEORY OF GENERAL RELATIVITY

LECTURE AND TUTORIALS – PROF. DR. HAYE HINRICHSSEN / M.SC. ALEXANDRE ALVAREZ – WS 2019/20



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EXERCISE 6.1: HODGE- \star OPERATION APPLIED TWICE (4P)

Let α be a p -form in n dimensions and let $s = \text{sign}(g)$. Show that

$$\star \star \alpha = s(-1)^{p(n-p)} \alpha$$

EXERCISE 6.2: DIRECTIONAL DERIVATIVE (5P)

In a two-dimensional space with coordinates $\{x, y\}$ let us consider the parametrized curve

$$c(\lambda) = \{\cos \lambda, 2 \sin \lambda\}.$$

- Find a representation of the tangent vector $\mathbf{X} = X^\mu \partial_\mu$ in these coordinates. (1P)
- Let us now consider a different coordinate system with the coordinates $\{a, b\}$, defined by the coordinate transformation $a = x + y$ and $b = \exp(x - y)$. Find the representation of the tangent vector \mathbf{X} in these coordinates. (2P)
- Find a coordinate system $\{u, v\}$ in which the vector field $\mathbf{Y} = \partial_x + x\partial_y$ is represented by $\mathbf{Y} = \partial_u$. (2P)

EXERCISE 6.3: COMPLEX DIFFERENTIAL (3P)

Let $z = re^{i\phi}$. Compute $\frac{1}{z}(dz \wedge d\bar{z})$ in terms of dr and $d\phi$.

($\Sigma = 12P$)

To be handed in on Wednesday, November 27, at the beginning of the tutorial.