

THEORY OF GENERAL RELATIVITY

LECTURE AND TUTORIALS – PROF. DR. HAYE HINRICHSSEN / M.SC. ALEXANDRE ALVAREZ – WS 2019/20

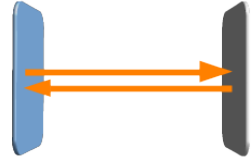
Warmup Exercise

EXERCISE 1.1: CONFINED ENERGY BEHAVES LIKE MASS

(2P)

A light ray with the total energy E is reflected infinitely many times without loss between two ideal massless mirrors.

- Compute the total 4-momenta p_R^μ and p_L^μ of the photons moving to the right and moving to the left.
- Show that the total system has an effective non-zero mass (despite the fact that the individual photons are massless).



EXERCISE 1.2: LIE ALGEBRA AND LIE GROUPS

(4P)

Consider a Lie algebra with three generators represented by

$$M = i(x\partial_y - y\partial_x), \quad P_x = -i\partial_x, \quad P_y = -i\partial_y$$

acting on functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}$:

- Compute all commutation relations (Lie algebra). (1P)
- Show that the exponential of M applied to a function can be expressed as

$$\exp(i\phi M)f(\vec{x}) = f(\mathbf{R}_\phi\vec{x}), \quad \phi \in [0, 2\pi]$$

where \mathbf{R}_ϕ is a 2×2 matrix. Calculate this matrix explicitly.

- Give an interpretation of the transformation $\exp(i\phi M)$ and $\exp(i\alpha P_x)$.

($\Sigma = 6P$)

To be handed in on Wednesday, October 23, at the beginning of the tutorial. Normally we have 12P per sheet. This week we only have a warmup exercise with 6P.